Isotropic versus anisotropic modeling of photorefractive solitons

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The question of the isotropic versus anisotropic modeling of incoherent spatial screening solitons in photorefractive crystals is addressed by a careful theoretical and numerical analysis. Isotropic, or local, models allow for an extended spiraling of two interacting scalar solitons, and for a prolonged propagation of vortex vector solitons, whereas anisotropic, nonlocal, models prevent such phenomena. In the context of Kukhtarev's material equations, the difference in behavior is traced to the continuity equation for the current density. We further show that neither an indefinite spiraling of two solitons nor stable propagation of vortex vector solitons is generally possible in both isotropic and anisotropic models. Such systems do not conserve angular momentum, even in the case of an isotropic change in the index of refraction.

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I. INTRODUCTION

For a few years now, the question of what exactly two interacting incoherent spatial-screening solitons do as they propagate down a photorefractive strontium barium niobate (SBN) crystal has occupied the attention of a number of researchers [1–10]. Although the question is of little relevance to perceived applications of photorefractive (PR) solitons, it is important in understanding the physics of the interaction between them.

One group of researchers claims that the PR screening solitons and the interaction between them is primarily isotropic in nature, even though the PR medium is inherently anisotropic [1]. Circular PR screening solitons are observed [2], and approximately described by isotropic local models [3]. Under proper initial conditions, two solitons spiral about each other in elliptical orbits [4]. The solitons behave similar to an attractive celestial two-body system that conserves angular momentum [5].

The other group of researchers claims that the PR material response is both anisotropic and a nonlocal function of the light intensity [6]. Strong anisotropy does not allow for radially symmetric solutions, hence circular solitons do not exist [7]. The interaction between solitons is anomalous in that they experience both attractive and repulsive forces [8]. This feature prevents stable spiraling, and causes beams to oscillate about each other and eventually fuse [9].

It is interesting to note that such varied, yet apparently valid, statements can be given about essentially the same physical system, described (by both groups) using the same, Kukhtarev model for the PR effect, and studied under similar experimental circumstances. The validity of claims by the first group stems primarily from experimental evidence, which is supported by theoretical modeling. The validity of claims by the second group is established by theoretical analysis, which is supported experimentally. Theoretical differences are the result of different levels of approximation applied to the standard Kukhtarev model. The difference in experimentation is contained in the fine print describing experimental conditions.

In one of our earlier publications [10] we pointed out how small changes in initial experimental conditions can blend one point of view (oscillation of solitons) with another (prolonged spiraling). In the present paper we address the question that seems to be at the core of the problem: How well, if at all, can the PR screening solitons be described by the local isotropic models? Modeling is important for spatial solitons because of the difficulties in observing solitons inside the crystal. On the matter of principle, the answer to the question posed is easy to give: not at all. PR screening solitons are inherently anisotropic. Yet, from the practical point of view, the initial stages of the interaction can be approximated by isotropic models, and both the isotropic and anisotropic models yield similar behavior. It matters little if the difference between the models becomes perceptible after 10 cm propagation through the crystal.

Recently, however, novel types of spatial solitary waves were proposed [11,12] and observed [12,13] in an SBN crystal—the vortex, the dipole-mode, and the propeller vector solitons—whose behavior is even more strikingly different in the two models than that of the spiraling solitons. They arise when a system of two beams is coaxially launched into the crystal, a fundamental beam in the form of a simple Gaussian and a higher order mode: a vortex beam or a dipole. In the course of joint propagation they form self-trapped vector solitons. In fact, the present situation is a bit confusing in that, on the level of theory, only local isotropic models are proposed and, on the level of experiment, only

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nonlocal anisotropic PR crystals are used [14,15].

The vortex and the dipole-mode vector solitons are introduced and discussed numerically in [11], and in [13] they are observed experimentally as well as studied numerically. It is found that the vortex vector solitons (with unit topological charge) are linearly unstable, decaying into the dipole-mode vector solitons, which apparently are stable. The two arms of the dipole-mode component are π phase shifted, and their dynamics resembles that of the two spiraling beams. Rotating propeller solitons are demonstrated experimentally and theoretically in [12]. They are the dipole vector solitons that rotate (both in intensity and phase) about the z axis.

We will show that the vortex vector solitons in isotropic models can propagate for tens of diffraction lengths before decaying into dipole-mode vector solitons, due to the transverse modulational instabilities and radiation losses. The resulting two beams of the dipole mode spiral about each other clockwise or counterclockwise, depending on the sign of the initial topological charge. As a result of further radiation loss and beam interaction, the angular momentum of the system, as well as the integrated power, slowly decreases.

On the other hand, the vortex vector solitons in anisotropic models are absolutely unstable, decaying into dipole-mode vector solitons within a fraction of diffraction length. The dynamics of the resulting two beams is completely different from the isotropic case. During the breakup phase the fragments of the vortex stay arrested oblique to the direction of the external electric field, and start rotating only after the formation of the dipole-mode soliton is complete. However, the two beams do not spiral about each other, but oscillate about the stable direction perpendicular to the external field. Correspondingly, the angular momentum of the system oscillates about zero. The oscillations are damped, and in the end a stable stationary dipole-mode vector soliton is formed, consisting of an elongated fundamental beam and a dipole perpendicular to the direction of the external field, with a π phase shift between the two beam arms.

The layout of this paper is as follows. Kukhtarev's material equations are introduced in Sec. II, the isotropic and the anisotropic models are defined in Sec. III, and the case of the spiraling scalar solitons is presented in Sec. IV. The propagation of vortex vector solitons is discussed in Sec. V, and the question of the nonconservation of angular momenta is addressed in Sec. VI. Section VII presents conclusions.

II. MATERIAL EQUATIONS

We introduce the models by analyzing Kukhtarev's material equations, and pinpoint the source of the trouble—the continuity equation for the current density. We integrate both models for exactly the same initial and boundary conditions, and display how the differences evolve. A surprising conclusion is that neither the isotropic model of spiraling solitons, nor the decaying vortex vector modes support stable spiraling, due to the modulational instabilities, and losses to radiation in the course of energy exchange between the beams. These effects further cause the power loss and the nonconservation of angular momentum.

The starting points are the standard Kukhtarev's equations for the generation/recombination rate of the mobile charges [16],

$$G = S_i(N_D - N_D^+)(I_b + I) - \gamma_R N_D^+ n, \tag{1}$$

the Poisson equation for the charge density,

$$\nabla \cdot \mathbf{D} = q(N_D^+ - N_A - n), \tag{2}$$

and the continuity equation,

$$q\partial_t n = qG + \nabla \cdot \mathbf{J}. \tag{3}$$

Here S_i is the cross section for photoexcitation, γ_R is the recombination rate, N_D, N_D^+, N_A, and n are the densities of donors, ionized donors, acceptors, and mobile charges, respectively. I_b is the background light intensity, I is the soliton intensity, $\mathbf{D} = \varepsilon \mathbf{E}$ is the space-charge displacement field (ε is the dielectric constant), and $\mathbf{J} = q\mu n\mathbf{E} + k_B T \nabla n$ is the current density (q and μ are the charge and mobility of the carriers, k_B is the Boltzmann's constant, and T is the temperature). To simplify bookkeeping, we assume $T=0$. This amounts to neglecting diffusion, which is not essential for our argument. G measures the rate of change of ionized donors, $G = \partial_t N_D^+$. We assume that the generation/recombination process reaches equilibrium much faster than the other processes in the crystal, which means $G=0$ (also a common assumption).

To treat the continuity equation, one has to find an expression for n. This is accomplished by combining the rate equation and the Poisson equation:

$$n = n_I \frac{N_D - N_A - n_{sc} - n}{N_A + n_{sc} + n}, \tag{4a}$$

where $n_I = S_i(I_b + I)/\gamma_R$ is the fraction of the mobile charge density directly proportional to the light intensity, and $n_{sc} = \nabla \cdot \mathbf{D}/q$ is the density of charge carriers giving rise to the space charge displacement field. This implicit equation can either be solved for n directly (as a quadratic equation) or iteratively, by substituting the zeroth-order approximation $n^{(0)} = n_I(N_D - N_A)/N_A$ into the right-hand side of Eq. (4a), and continuing the process. In the first order, for example, one obtains a saturable model [17] $n^{(1)} = n_I(N_D - N_A)N_A/(N_A^2 + n_I N_D)$. In typical PR crystals, under conditions presumed for the generation of screening solitons, it is $N_D, N_A >> n_{sc}, n_I$, so that to a good approximation $n \approx n^{(0)}$. Hence, approximately n is directly proportional to $I_b + I$:

$$n \approx \frac{S_i(N_D - N_A)}{\gamma_R N_A}(I_b + I). \tag{4b}$$

This formula is often explored in the modeling.

III. MODELS

The solution of the steady-state continuity equation

$$\nabla \cdot (n\mathbf{E}) = 0 \tag{5}$$

crucially depends on whether one- (transverse) dimensional (1D) or two-dimensional (2D) solitons are considered. In 1D one has the equation $\partial_x(nE)=0$, which yields $nE=n_\infty E_0$, where E_0 is the externally applied field and n_∞ is the charge density at transverse infinity. This leads to the standard isotropic 1D model [18] for the space-charge field,

$$E=E_0\frac{I_b+I_\infty}{I_b+I},\tag{6}$$

where I_∞ is the beam intensity at infinity. In 2D, the continuity equation has the form

$$\partial_x(nE_x)+\partial_y(nE_y)=0,\tag{7}$$

and the isotropic model cannot be the general solution. It requires that both terms in Eq. (7) equal 0, whereas the equation only requires that they should balance each other out. One can choose it as a special solution, and see how consistent it is with the general development. We choose the simplest possibility, $E_y=0$, to compare easily with the general anisotropic model. Other choices are possible [3,19], and all of them are equally good for the initial stages of the process. However, all of the isotropic models fail equally after some propagation in the crystal.

With the choice $E_y=0$ and E_x, as in Eq. (6), one proceeds to solve the paraxial propagation equation for the beam envelope

$$2ikn_0\partial_z A+\nabla^2 A=-k^2n_0^4r_{33}EA,\tag{8}$$

where k is the wave number in vacuum, n_0 is the bulk refractive index, and r_{33} is the component of the electro-optic tensor that couples to the space-charge field. The beam propagates in the z direction, and is polarized along the x direction, which is also the direction of the crystalline c axis.

In the anisotropic model one makes no *a priori* ansatz, but generally proceeds to solve Eq. (5) by introducing an electrostatic potential $\nabla\phi=\mathbf{E_0}-\mathbf{E}$ that takes care of the boundary conditions [20]. Upon substituting the solution from Eq. (4b), into Eq. (5) one obtains

$$\nabla^2\phi+\nabla\phi\cdot\nabla\ln(1+I)=E_0\partial_x\ln(1+I),\tag{9}$$

where the intensity is now normalized to I_b. This equation has to be solved numerically, together with the paraxial propagation equation appropriate to the anisotropic model:

$$2ikn_0\partial_z A+\nabla^2 A=-k^2n_0^4r_{33}(E_0-\partial_x\phi)A.\tag{10}$$

In this manner, the two models are defined: the isotropic by Eqs. (6) and (8), and the anisotropic by Eqs. (9) and (10). In the case of multicomponent beams one has to solve the system of equations for all of the components, and I in the equations then stands for the total intensity. We proceed to integrate and compare the models for the two characteristic cases.

It should be noted that the integration of both models is carried out using two independent integration methods. They produced, qualitatively and quantitatively, very similar results.

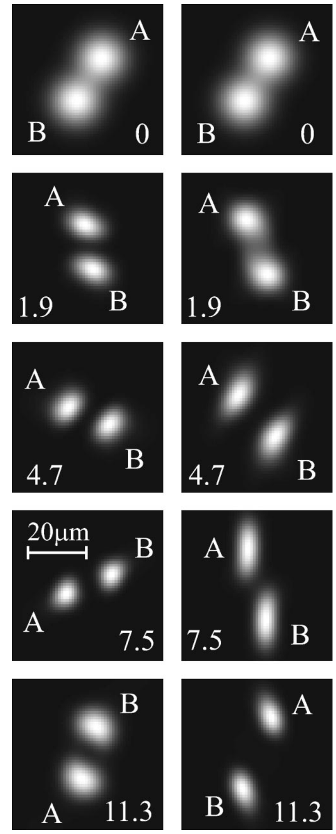

FIG. 1. Two solitons (A and B) propagating in a SBN crystal, shown at different propagation depths. Left column, the isotropic model; right column, the anisotropic model. Propagation distances in mm are noted in each of the figures. Other parameters are: The initial intensity of the beams is $I_0=5$, the initial width $w=13$ μm, and $E_0=4$ kV/cm.

IV. SPIRALING SCALAR SOLITONS

First, we consider the case of two spiraling incoherent scalar solitons. We integrate both models for exactly the same set of parameters, found in the experimental part of Ref. [19]. The steady-state transverse intensity distribution of the two interacting solitons at different depths within the crystal is shown in Fig. 1. As expected, the isotropic model leads to prolonged spiraling, whereas the anisotropic model precludes such a phenomenon. However, up to a thickness of ~ 5 mm the two models produce similar behavior. The beams rotate for $\sim\pi/2$ in both cases, but then the anisotropic solitons stop spiraling and start oscillating. It is interesting to note that in the exit crystal face both models produced similar beam distributions; however, the beam positions are inverted.

Figure 2 represents the space-charge distributions corresponding to Fig. 1. In the isotropic case, only E_x is shown, since the other component remains zero. The space-charge distribution is merely the negative replica of the intensity distribution, imprinted on the background field E_0. This is not so for the anisotropic model. Both components there are nonzero, and show considerably more structure than the isotropic model. The E_x component exhibits features that are both above (bright regions in the figure) and below (dark regions) the level of the external field. The positive regions

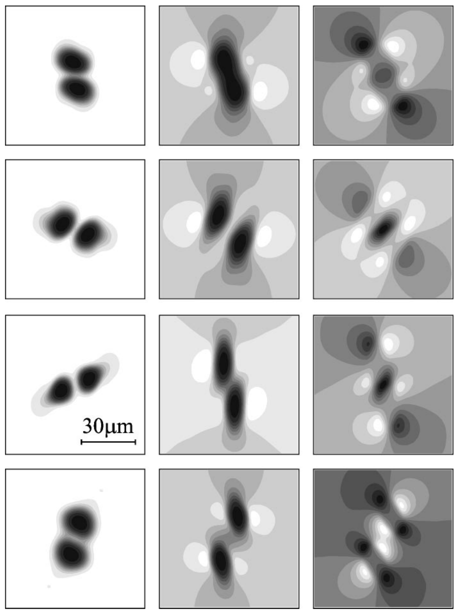

FIG. 2. Distribution of the space-charge field, corresponding to Fig. 1 (initial distribution omitted). Left column, the isotropic model; middle column, E_x; and right column, E_y of the anisotropic model.

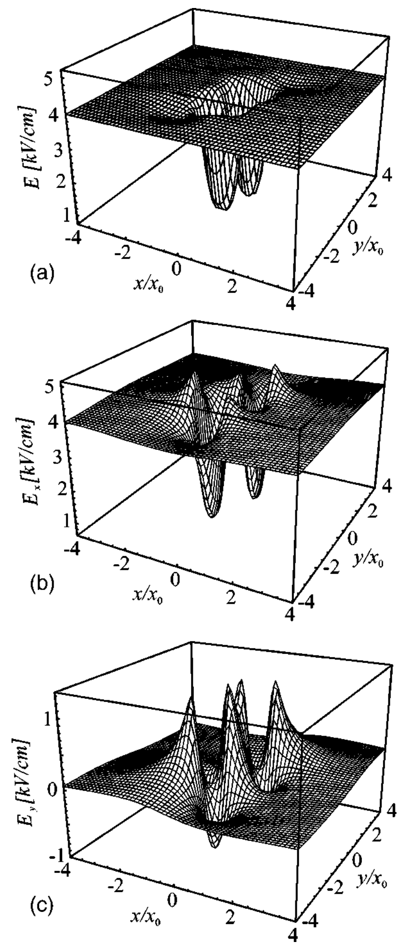

FIG. 3. Surface plots of the components of the space-charge field from Fig. 2 at $z=11.3$ mm. (a) E_x of the isotropic model; and (b) E_x and (c) E_y of the anisotropic model.

are crucial in understanding the existence of repulsive forces between solitons [8], and are responsible for preventing the spiraling. There are no positive regions in the space-charge field of the isotropic model (no negative intensity), hence it cannot explain the existence of repulsive forces. It only predicts attractive forces between solitons, and can only lead to gravitational analogies. For both models, surface plots of the space-charge field at the exit face are presented in Fig. 3.

The y component of the anisotropic space-charge field also consists of positive and negative parts, and is predominantly a multipole electric field in nature (quadrupole and higher). The E_x component is predominantly a dipole field. Even though the E_y field does not influence the propagation of the solitons much (it does not couple to the effective component of the electro-optic tensor), it is important for the overall picture, as a part of the general solution of the continuity equation. It affects the form of the space-charge field, and contributes to the nonlocal, anisotropic effects in the interaction of solitons.

V. VORTEX VECTOR SOLITONS

Next, we consider the propagation of vortex vector solitons. Again, a system of two beams, the fundamental Gaussian and a vortex of the form $A_0(\rho/w)^2\exp(-\rho^2/w^2)\exp(i\varphi)$, where (ρ,φ) are the polar coordinates in the transverse (x,y) plane, is launched into the crystal and propagated using the isotropic and the anisotropic codes, for similar sets of parameters. Figure 4 depicts the transverse beam distributions in the crystal at various depths. It is seen that the isotropic vortex exhibits pronounced stability, apparently not changing for more than 20 diffraction lengths, which corresponds to ~ 100 mm propagation through the crystal. As the instability sets in, two lobes develop at the opposite sides of the distribution, rotating about the rim. The vortex elongates and decays into two filaments that vigorously spiral about each other, similarly to the propeller solitons. The phase difference, which was running linearly from 0 to 2π along the vortex, is exactly π between the filaments.

In the case of the anisotropic model, a different scenario unfolds. The vortex beam develops a modulational instability within a fraction of diffraction length, and breaks into two elongated beams. During the breakup process, the fragments are almost stationary, and then start rotating toward the equilibrium position, which is in the direction perpendicular to the external field. In the end, they oscillate about the stable solution—the dipole-mode soliton. It should be noted that the vortices, both isotropic and anisotropic, can break into more than two fragments depending on the values of parameters, the width of the doughnut mode, and the vortex charge. In such a case, the higher-order stable modes—the tripole, quadrupole, etc.—can be reached.

It appears that these findings are in agreement with the experimental findings in [13].

VI. NONCONSERVATION OF ANGULAR MOMENTUM

An interesting question to ask is what happens to the angular momentum of the system as it evolves? On theoretical

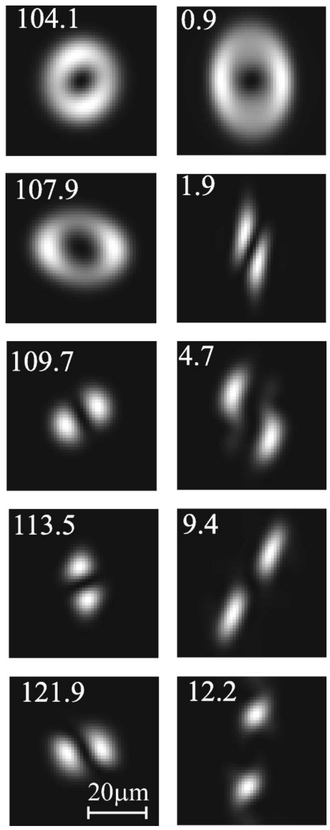

FIG. 4. Breakup during propagation of the vortex component of a vector soliton. Left column, the isotropic model; right column, the anisotropic model. Even though the breakup phases appear similar, note the widely different propagation distances in each of the figures. Initial intensities of the fundamental (not shown) and of the vortex component were $I_0=9$, and other parameters are as in Fig. 1.

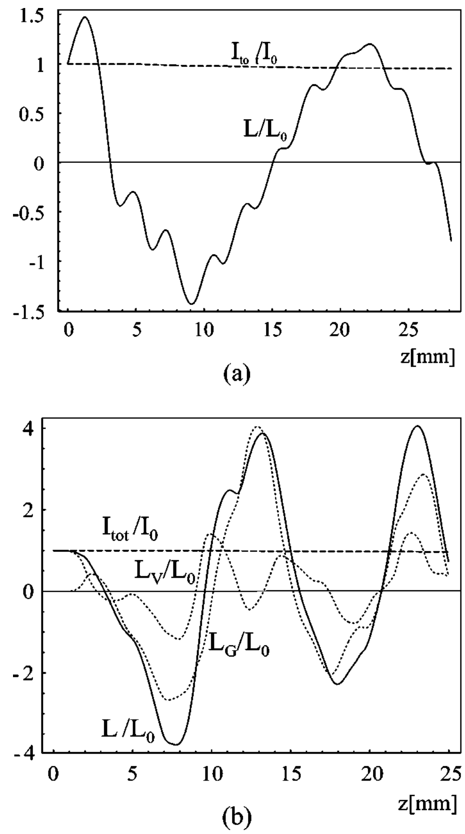

FIG. 5. (a) Angular momenta of the pair of anisotropic solitons from Fig. 1, and (b) of the anisotropic vortex vector soliton from Fig. 4, as functions of the crystal thickness. Dashed lines represent the total intensity. Dotted lines represent the momenta of the vortex and of the fundamental component.

grounds, being the consequence of the isotropy of space, the momentum should be constant for the isotropic model, and it should vary for the anisotropic model. Figures 5 and 6 depict the situation.

In the case of the spiraling anisotropic solitons [Fig. 5(a)], or the vortex vector soliton [Fig. 5(b)], the situation is as expected. Initially the momenta of both models are close; however, upon approaching each other the solitons start exchanging energy. The spiraling solitons arrest mutual rotation and reverse its sense, and start to oscillate about the stable direction perpendicular to the external field. The momentum reverses its sign periodically and performs damped oscillations about zero. It is similar for the vortex vector soliton: after a few mm of propagation, the vortex component breaks up, and the momentum drops to zero and reverses the sign. As a result of the interaction between components, the fundamental beam acquires a nonzero momentum, which initially is in counterphase to the dipole momentum. However, this relation is soon violated, and the momenta do not balance each other out. The total angular momentum keeps oscillating about zero. A similar scenario unfolds for various multipole vector solitons.

The situation with the isotropic model is somewhat unexpected in that the angular momentum is also not conserved. The isotropic spiraling solitons keep spiraling, but their angular momentum is not constant [Fig. 6(a)]. It changes due to a strong energy exchange, which also occurs in isotropic models [19]. As the solitons start interacting, part of the total energy is lost to radiation. A part of each soliton is trapped by the other, and the identity of individual solitons becomes questionable. Consequently, a part of the momentum is taken by the radiation, and the momentum of the solitons slowly drops. Hence, even the isotropic solitons do not support indefinite spiraling. In a generic situation, with Gaussian beams launched and with reasonable initial momenta supplied to the beams, one can observe spiraling only over finite distances (which may exceed the thicknesses of commonly available crystals). On the other hand, for anisotropic beams the spiraling is suppressed and ceases after a few mm of propagation.

A similar picture holds for the total angular momentum of the isotropic vortex vector soliton [Fig. 6(b)], although the momenta of components vary considerably. The angular momentum remains constant for as long as the integrity of the vortex vector soliton is intact. However, as the vortex breaks up, the system starts radiating and the momenta of components start oscillating. The total momentum slowly declines, but remains positive. The changes in the momenta of the fundamental and dipole beams oscillate in counterphase more closely than in the anisotropic case, but they do not exactly cancel each other out.

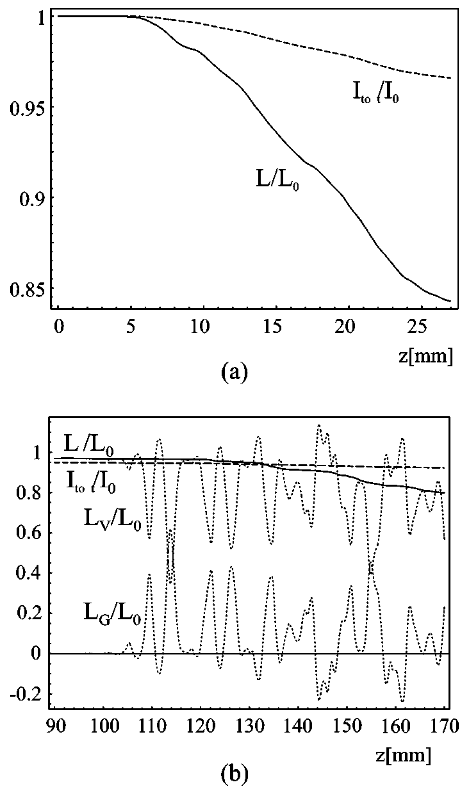

FIG. 6. (a) Angular momenta of the isotropic spiraling solitons, and (b) of the isotropic vortex vector soliton from Figs. 1 and 4, as functions of the crystal thickness. Dashed lines represent the total intensity. Dotted lines represent the momenta of the vortex and of the fundamental component.

A part of the mystery of nonconservation of angular momentum in the isotropic model lies in a rather precipitous drop in the momentum, either when the strong interaction between the spiraling solitons occurs, or after the vortex breakup. Such a drop occurs at the place where the system strongly radiates; however, the radiation loss accounts for a small change in the total power or energy, as is visible in Figs. 6(a) and 6(b), and it cannot be expected to cause such a large change in the momentum. According to theory [14,21], angular momentum should be strictly proportional to the total beam power or energy. We should mention that this feature is numerically robust, occurring in all of our computational routines. We believe that, in addition to radiation losses, the source of momentum loss is the strong overlap, or the induced coherence [19], or the "mass" (energy) exchange [9,10] between the system components, as they interact. There is no reason to believe that angular momentum, or any other of the "conserved quantities," should be conserved in a nonintegrable system such as ours.

VII. CONCLUSIONS

The differences between the isotropic, or local, and anisotropic, or nonlocal, modeling of spatial solitons in photorefractive crystals have been considered in detail. On the level of Kukhtarev's material equations, the difference in behavior of interacting solitons is traced to the continuity equation for the current density. Both the isotropic and anisotropic models are solved concurrently for identical sets of parameters and for a few characteristic examples of soliton propagation: the spiraling of incoherent solitons, and the breakup of vortex vector solitons.

It is found that the two models offer widely different pictures of soliton behavior, even though the initial stages of soliton propagation look similar. The isotropic model allows for an extended spiraling of two incoherent solitons and for a prolonged propagation of vortex vector solitons. When the vortex eventually breaks, the fragments continue to spiral about each other. The anisotropic model prevents such phenomena. The spiraling of solitons stops after a few turns, and the beams start to oscillate about the stable direction perpendicular to the external field. The vortex vector soliton breaks after only a few mm of propagation, and the fragments are arrested again by the stable perpendicular direction.

The differences between the models persist even when one considers the quantities that are normally conserved during solitonic propagation, such as the integrated power or the angular momentum of the system. Due to the interaction and instability of beams, the system radiates, and the radiation takes away part of the power and angular momentum. In addition, the "mass" (energy) exchange between the beam components affects the "conservation" laws in a system that is not integrable. Hence, even the angular momentum of the isotropic model is not conserved. On the other hand, the anisotropic model represents a noncentral mechanical system for which there is no reason for the angular momentum to be conserved in the first place. It drops rapidly, and performs damped oscillations about zero.

We conclude that, although allowed, isotropic models impose constraints on the general description of 2D photorefractive screening solitons that are not warranted from the physical point of view. They lead to a simplified form of the space-charge field that cannot capture all of the important features (such as repulsion) observed in the behavior of interacting solitons. They should only be used for short propagation distances and qualitative purposes.

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